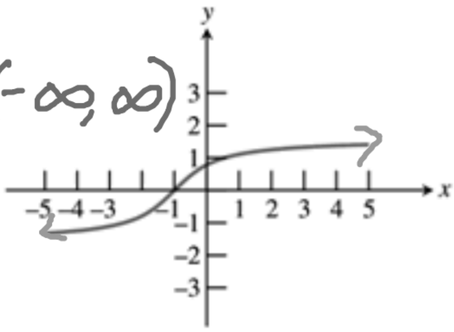
Warm Up-~~Identify the domain and range of each using interval notation~~.
Then identify increasing, decreasing, and/or constant intervals.

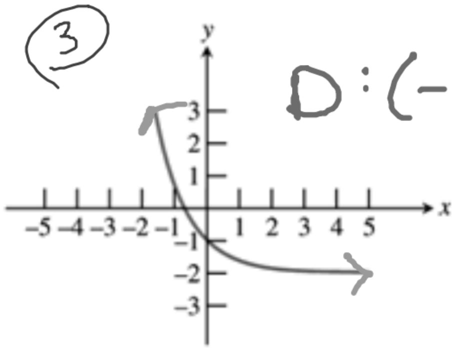
1

I: $(-\infty, \infty)$



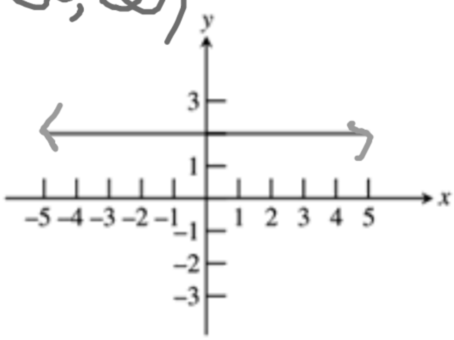
3

D: $(-\infty, \infty)$

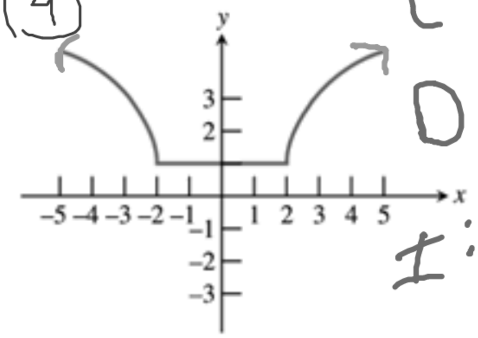


2

C: $(-\infty, \infty)$



4



C: $[-2, 2]$

D: $(-\infty, -2]$

I: $[2, \infty)$

WARM UP:

1. Find a partner.
2. Grab a pair of scissors.
3. Grab two worksheets on the table.
4. Wait for directions...

Section 1.5: Inverse Relations.

Inverse Relation

The ordered pair (a,b) is in a relation if and only if the pair (b,a) is in the **inverse relation**.

"switch x and y"

Inverse Function

$f^{-1}(x)$

y^{-1}

If f is a one-to-one function with domain D and range R, then the **inverse function of f**, denoted f^{-1}, is the function with domain R and range D defined by $f^{-1}(b) = a$ if and only if $f(a) = b$.

Example **Finding an Inverse Function** $\boxed{\textbf{Algebraically}}$

$f^{-1}(x)$

Find an equation for $f^{-1}(x)$ if $f(x)=\dfrac{2x}{x-1}$.

$x=\dfrac{2y}{y-1}$ Switch the x and y

Solve for y:

$x(y-1)=2y$ Multiply by $y-1$

$xy-x=2y$ Distribute x

$xy-2y=x$ Isolate the y terms

$y(x-2)=x$ Factor out y

$y=\dfrac{x}{x-2}$ Divide by $x-2$

Therefore $f^{-1}(x)=\dfrac{x}{x-2}$.

$$y=\frac{2x}{x-1}$$

$$(y-1)x=\frac{2y}{\cancel{(y-1)}}\cdot\frac{\cancel{(y-1)}}{1}$$

$$xy-x=2y$$

$$xy-2y=x$$

$$\frac{y\cancel{(x-2)}}{\cancel{(x-2)}}=\frac{x}{(x-2)}$$

$$\boxed{y^{-1}=\frac{x}{x-2} \quad \text{OR} \quad f^{-1}(x)=\frac{x}{x-2}}$$

The Inverse Composition Rule

A function f is one-to-one with inverse function g if and only if
$f(g(x)) = x$ for every x in the domain of g, and
$g(f(x)) = x$ for every x in the domain of f.

Example **Verifying Inverse Functions**

Show algebraically the $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ are inverse functions.

Use that Inverse Composition Rule:

$f(g(x)) = f(\sqrt[3]{x-2}) = \left(\sqrt[3]{x-2}\right)^3 + 2 = x - 2 + 2 = x$

$g(f(x)) = g(x^3 + 2) = \sqrt[3]{(x^3+2) - 2} = \sqrt[3]{x^3} = x$

Since these equations are true for all x, f and g are inverses.

Ex) Verify that $f(x) = \frac{7}{x}$ and $g(x) = \frac{7}{x}$ are inverses.

$$f(g(x)) = f\left(\frac{7}{x}\right) = \frac{7}{\frac{7}{x}} = \frac{\frac{7}{1}}{\frac{7}{x}}$$

$$\frac{\cancel{7}}{1} \cdot \frac{x}{\cancel{7}} = x$$

$$g(f(x)) = g\left(\frac{7}{x}\right) = \frac{7}{\frac{7}{x}} = \frac{7}{1} \cdot \frac{x}{7} = \frac{\cancel{7}x}{\cancel{7}} = x$$

f and g are inverses!

In Exercises 13–22, find a formula for $f^{-1}(x)$.

13. $f(x) = 3x - 6$

14. $f(x) = 2x + 5$

15. $f(x) = \dfrac{2x - 3}{x + 1}$

~~**16.**~~ $f(x) = \dfrac{x + 3}{x - 2}$

17. $f(x) = \sqrt{x - 3}$

18. $f(x) = \sqrt{x + 2}$

In Exercises 27–32, confirm that f and g are inverses by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

27. $f(x) = 3x - 2$ and $g(x) = \dfrac{x + 2}{3}$

28. $f(x) = \dfrac{x + 3}{4}$ and $g(x) = 4x - 3$

29. $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$

30. $f(x) = \dfrac{7}{x}$ and $g(x) = \dfrac{7}{x}$

31. $f(x) = \dfrac{x + 1}{x}$ and $g(x) = \dfrac{1}{x - 1}$

32. $f(x) = \dfrac{x + 3}{x - 2}$ and $g(x) = \dfrac{2x + 3}{x - 1}$

Exit Quiz/ticket

Given $h(x) = \sqrt{2x+12}$. What is the domain and range of the function? Express in interval notation.

13. $y = 3x - 6 \Rightarrow$
$$\begin{aligned} x &= 3y - 6 \\ 3y &= x + 6 \\ f^{-1}(x) = y &= \frac{x+6}{3} = \frac{1}{3}x + 2; \end{aligned}$$

14. $y = 2x + 5 \Rightarrow$
$$\begin{aligned} x &= 2y + 5 \\ 2y &= x - 5 \\ f^{-1}(x) = y &= \frac{x-5}{2} = \frac{1}{2}x - \frac{5}{2}; \end{aligned}$$

15. $y = \dfrac{2x-3}{x+1} \Rightarrow$
$$\begin{aligned} x &= \frac{2y-3}{y+1} \\ x(y+1) &= 2y - 3 \\ xy + x &= 2y - 3 \\ xy - 2y &= -x - 3 \\ y(x-2) &= -(x+3) \\ f^{-1}(x) = y &= -\frac{x+3}{x-2} = \frac{x+3}{2-x}; \end{aligned}$$

16. $y = \dfrac{x+3}{x-2} \Rightarrow$
$$\begin{aligned} x &= \frac{y+3}{y-2} \\ x(y-2) &= y + 3 \\ xy - 2x &= y + 3 \\ xy - y &= 2x + 3 \\ y(x-1) &= 2x + 3 \\ f^{-1}(x) = y &= \frac{2x+3}{x-1}; \end{aligned}$$

17. $y = \sqrt{x-3},\ x \geq 3,\ y \geq 0 \Rightarrow$
$$\begin{aligned} x &= \sqrt{y-3}, \\ x^2 &= y - 3, \\ f^{-1}(x) = y &= x^2 + 3, \end{aligned}$$

18. $y = \sqrt{x+2},\ x \geq -2,\ y \geq 0 \Rightarrow$
$$\begin{aligned} x &= \sqrt{y+2}, \\ x^2 &= y + 2, \\ f^{-1}(x) = y &= x^2 - 2, \end{aligned}$$

27. $f(g(x)) = 3\left[\frac{1}{3}(x+2)\right] - 2 = x + 2 - 2 = x;$

$g(f(x)) = \frac{1}{3}[(3x-2)+2] = \frac{1}{3}(3x) = x$

28. $f(g(x)) = \frac{1}{4}[(4x-3)+3] = \frac{1}{4}(4x) = x;$

$g(f(x)) = 4\left[\frac{1}{4}(x+3)\right] - 3 = x + 3 - 3 = x$

29. $f(g(x)) = [(x-1)^{1/3}]^3 + 1 = (x-1)^1 + 1$

$= x - 1 + 1 = x;$

$g(f(x)) = [(x^3+1)-1]^{1/3} = (x^3)^{1/3} = x^1 = x$

30. $f(g(x)) = \dfrac{7}{\frac{7}{x}} = \frac{7}{1}\cdot\frac{x}{7} = x;\ g(f(x)) = \dfrac{7}{\frac{7}{x}} = \frac{7}{1}\cdot\frac{x}{7} = x$

31. $f(g(x)) = \dfrac{\frac{1}{x-1}+1}{\frac{1}{x-1}} = (x-1)\left(\frac{1}{x-1}+1\right)$

$= 1 + x - 1 = x;$

$g(f(x)) = \dfrac{1}{\frac{x+1}{x}-1} = \left(\dfrac{1}{\frac{x+1}{x}-1}\right)\cdot\frac{x}{x}$

$= \dfrac{x}{x+1-x} = \dfrac{x}{1} = x$

32. $f(g(x)) = \dfrac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2} = \left(\dfrac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}\right)\cdot\left(\frac{x-1}{x-1}\right)$

$= \dfrac{2x+3+3(x-1)}{2x+3-2(x-1)} = \dfrac{5x}{5} = x;$

$g(f(x)) = \dfrac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}$

$= \left[\dfrac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}\right]\cdot\left(\frac{x-2}{x-2}\right)$

$= \dfrac{2(x+3)+3(x-2)}{x+3-(x-2)} = \dfrac{5x}{5} = x$