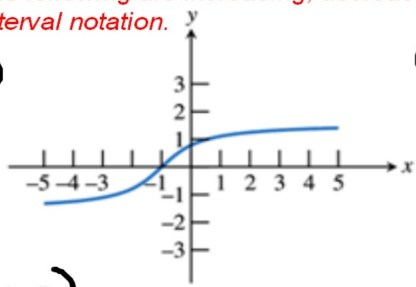
August 31 Warm Up - Increasing/Decreasing Functions

State whether the following are increasing, decreasing, or constant.
Then state in interval notation.

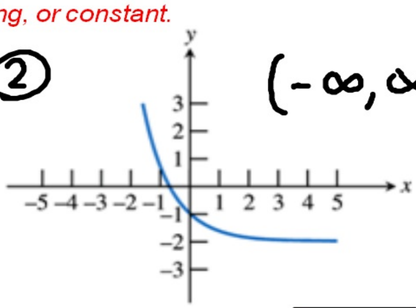
1



$(-\infty, \infty)$.

Increasing

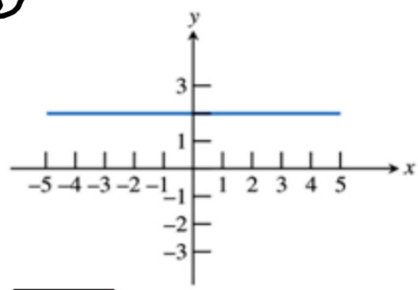
2



$(-\infty, \infty)$.

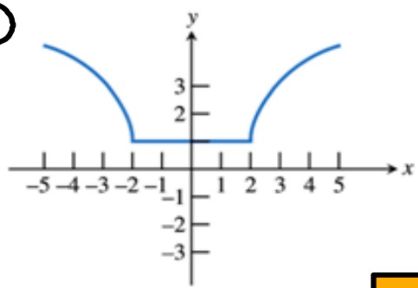
Decreasing

3



Constant

4



Decreasing on $(-\infty, -2]$
Constant on $[-2, 2]$
Increasing on $[2, \infty)$

Section 1.5: Inverse Relations.

Inverse Relation

The ordered pair (a,b) is in a relation if and only if the pair (b,a) is in the **inverse relation**.

$y = 2x$

$\frac{x}{2} = \frac{2y}{2}$

$y = \frac{1}{2}x$

$f^{-1}(x) = \frac{1}{2}x$

Inverse Function

If f is a one-to-one function with domain D and range R, then the **inverse function of f**, denoted f^{-1}, is the function with domain R and range D defined by $f^{-1}(b) = a$ if and only if $f(a) = b$.

Example **Finding an Inverse Function Algebraically**

Find an equation for $\cancel{f(x)}$ $f^{-1}(x)$ if $f(x) = \dfrac{2x}{x-1}$. $\Rightarrow y = \dfrac{2}{x-1}$

$x = \dfrac{2y}{y-1}$ Switch the x and y

Solve for y:

$x(y-1) = 2y$ Multiply by $y-1$

$xy - x = 2y$ Distribute x

* $xy - 2y = x$ Isolate the y terms

$y(x-2) = x$ Factor out y

$y = \dfrac{x}{x-2}$ Divide by $x-2$

Therefore $f^{-1}(x) = \dfrac{x}{x-2}$.

The Inverse Composition Rule

A function f is one-to-one with inverse function g if and only if
$f(g(x)) = x$ for every x in the domain of g, and
$g(f(x)) = x$ for every x in the domain of f.

Example **Verifying Inverse Functions**

Show algebraically the $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ are inverse functions.

$f(g(x)) = f(\sqrt[3]{x-2}) =$

Use that Inverse Composition Rule:

$f(g(x)) = f(\sqrt[3]{x-2}) = \left(\sqrt[3]{x-2}\right)^3 + 2 = x - 2 + 2 = x$

$g(f(x)) = g(x^3 + 2) = \sqrt[3]{(x^3+2)-2} = \sqrt[3]{x^3} = x$

Since these equations are true for all x, f and g are inverses.

~~Homework: p. 135~~: 13-18, 27-32,

Section 1.5

In Exercises 13–22, find a formula for $f^{-1}(x)$.

13. $f(x) = 3x - 6$

14. $f(x) = 2x + 5$

15. $f(x) = \dfrac{2x - 3}{x + 1}$

16. $f(x) = \dfrac{x + 3}{x - 2}$

17. $f(x) = \sqrt{x - 3}$

18. $f(x) = \sqrt{x + 2}$

In Exercises 27–32, confirm that f and g are inverses by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

27. $f(x) = 3x - 2$ and $g(x) = \dfrac{x + 2}{3}$

28. $f(x) = \dfrac{x + 3}{4}$ and $g(x) = 4x - 3$

29. $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$

30. $f(x) = \dfrac{7}{x}$ and $g(x) = \dfrac{7}{x}$

31. $f(x) = \dfrac{x + 1}{x}$ and $g(x) = \dfrac{1}{x - 1}$

32. $f(x) = \dfrac{x + 3}{x - 2}$ and $g(x) = \dfrac{2x + 3}{x - 1}$